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63-2-5

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Interim Research Memorandum

OPERATIONS EVALUATION GROUP

Center for Naval Analyses

WASHINGTON 25, D. C.

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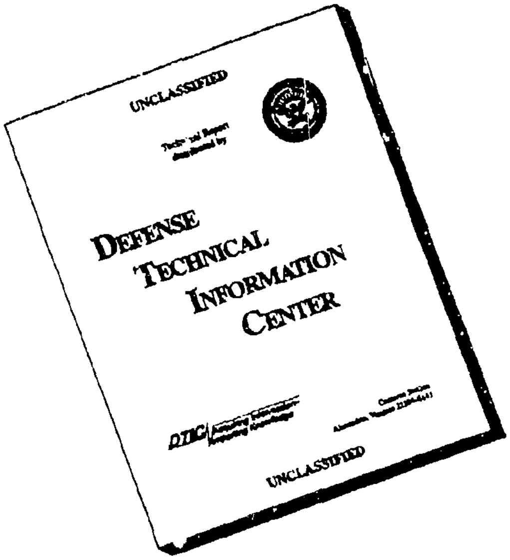

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9 November 1962

INTERIM RESEARCH MEMORANDUM

OPERATIONS EVALUATION GROUP

ESTIMATING THE LATERAL RANGE CURVE FROM OBSERVED DETECTION RANGES

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IRM-27

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ABSTRACT

The problem of estimating the distribution of detection ranges, e.g., of a sonar or radar, in which there is no information on the total number of detection opportunities or the target tracks, has been treated in OEG IRM-7, (reference (1)). A more detailed discussion of the assumptions and derivations is presented here; the result differs slightly from that given in IRM-7.

We are given an observer with a detection device, and a collection of numbers r_i, $i=1, \ldots, N$, representing the ranges at which targets of a certain type have been detected. It is assumed that the detection device has a range r, called the detection range, which varies randomly as time passes, but that it is essentially constant during the passage of any one target. A target is assumed to be detected, at range r, if and only if it passes within a range r of the detection device. The path of each target is a straight line. Only one target at a time passes by.

It is known that there are cases in which targets pass directly by the detector and go undetected. We may think of the detector as "not working" in such cases. It is easy to include this situation in our model; we simply define the detection range r to be $r=-1$ when the device is not working. If γ_o is the probability that the detection device is not working, then, of course, $r = -1$ with probability γ_o. We cannot estimate γ_o in the situation described here.

The main thing is the assumption that the detection range against the given type of target is a random variable associated with the detection device; it does not depend on the individual target, or indeed, on the presence or absence of a target. During the total time of the "experiment," N targets were detected, at ranges $r_1, \ldots, r_N$; we don't know how many targets passed by that were not detected.[1]

We shall denote by ρ the lateral range of any target (detected or not); this is the shortest distance from the observer to the straight line path traversed by the target, i.e., the "distance of closest approach." From the definition of the detection range, we see that a target is detected if and only if $o \leq \rho \leq r$. Both ρ and r are random variables; ρ is associated with the target, and r with the detection device. It follows that ρ are r are independent random variables.

In the following, r_o denotes an arbitrary, but fixed, positive number.

We are interested in two functions:

$$f(r_o) = \Pr(r \geq r_o), \tag{1}$$

and

$$L(r_o) = \Pr(\text{a target is detected, given that its lateral range is } r_o). \tag{2}$$

The first function $f(r_o)$ depends only on the detection device, but we can estimate it only through observed detections, i.e., targets are needed.

The probability space that is involved here is evidently the (ρ, r) plane, $\rho \geq o$, $r \geq -1$. The observed ranges $r_1, \ldots, r_N$ of the detected targets are independent samples of a certain probability distribution. This distribution is nothing more nor less than the distribution of detection ranges (of the detection device) given a (target) detection, i.e., the distribution of r given $o \leq \rho \leq r$, if we recall that a target is detected if and only if $o \leq \rho \leq r$.

Regarding (2), we can write

$$L(r_o) = Pr(o \leq \rho \leq r \mid \rho = r_o)$$

$$= Pr(o \leq r_o \leq r \mid \rho = r_o) .$$

But in the last expression, the two events $r_o \leq r$ and $\rho = r_o$ are independent. Hence

$$L(r_o) = Pr(r_o \leq r) = f(r_o) . \qquad (3)$$

by (1). This appears in [1] as Theorem 1.

So all we need now is a reasonable estimate for $f(r_o)$. Since we have plenty of samples of the distribution of r given detection, we can write

$$Pr(r_o \leq r < r_o + \Delta r_o \mid \text{detection}) \approx \frac{\Delta n}{N} , \qquad (4)$$

in which Δn is the number of observed detections with ranges in the interval $(r_o, r_o + \Delta r_o)$, $r_o \geq o$, and N is the total number of detections. But

$$\begin{aligned} &Pr(r_o \leq r < r_o + \Delta r_o \mid \text{detection}) \\ &= Pr(r_o \leq r < r_o + \Delta r_o \mid o \leq \rho \leq r) \qquad (5) \\ &= \frac{Pr(r_o \leq r < r_o + \Delta r_o \text{ and } \rho \leq r)}{Pr(\rho \leq r)} \end{aligned}$$

Let $\alpha = \Pr(\rho \le r) = \Pr(\text{detection})$.

Now the numerator of (5) differs only negligibly from

$$\Pr(r_0 \le r < r_0 + \Delta r_0 \text{ and } \rho \le r_0)$$

if Δr_0 is small, and provided that ρ has a continuous distribution. Since $\{r_0 \le r < r_0 + \Delta r_0\}$ and $\{\rho \le r_0\}$ are independent events, (4) and (5) now yield

$$\frac{\Pr(r_0 \le r < r_0 + \Delta r).\ \Pr(\rho \le r_0)}{\alpha} \approx \frac{\Delta n}{N},$$

or

$$\Pr(r_0 \le r < r_0 + \Delta r_0) \approx \frac{\alpha}{N} \quad \frac{\Delta n}{\Pr(\rho \le r_0)}. \tag{6}$$

Now let us suppose that ρ is uniformly distributed in an interval $0 \le \rho \le R$. Then (6) becomes

$$\Pr(r_0 \le r < r_0 + \Delta r_0) = \begin{cases} \dfrac{\alpha}{N} \quad \dfrac{R\,\Delta n}{r_0} & \text{if } r_0 < R, \\[2ex] = \dfrac{\alpha}{N}\,\Delta n & \text{if } r_0 \ge R. \end{cases} \tag{7}$$

In an unpublished working paper, J. Neuendorffer, who first considered this problem, groups his observed ranges into range bands. While his estimate is basically good, it contains an artificial bias that is introduced by the choice of end points for his range bands.

Here we shall simply take as our estimate for $f(r_0)$ — or rather $1 - f(r_0)$, which is nondecreasing — that discrete distribution which is determined by the right side of (7). That equation will be satisfied if we take the jumps in $1 - f(r)$ — we drop the subscript from r_0 — to be located in $r = -1$ and

$r = r_1, \ldots, r_N$, with magnitudes $\gamma_o, \gamma_1, \ldots$ in which

$$\left.\begin{aligned} \gamma_o &= \Pr(r = -1), \text{ unknown,} \\ \gamma_i &= \frac{\alpha}{N} \frac{R}{r_i} \quad \text{if } r_i \leq R, \\ \gamma_i &= \frac{\alpha}{N} \quad \text{if } r_i > R. \end{aligned}\right\} \tag{8}$$

By summing for $i = 1, \ldots, N$, we obtain

$$1 - \gamma_o = \frac{\alpha}{N} \left[\sum_{r_i \leq R} \frac{R}{r_i} + M \right], \tag{9}$$

in which M is the number of observed r_i larger than R. From (9), we can estimate α/N:

$$\alpha/N = \frac{1 - \gamma_o}{\sum_{r_j \leq R} \frac{R}{r_j} + M} \tag{10}$$

and (8) can be written

$$\left.\begin{aligned} \gamma_o &= \Pr(r = -1), \\ \gamma_i &= \frac{R}{r_i} \cdot \frac{1 - \gamma_o}{\sum_{r_j \leq R} \frac{R}{r_j} + M} \quad \text{if } r_i \leq R \\ \gamma_i &= \frac{1 - \gamma_o}{\sum_{r_j \leq R} \frac{R}{r_j} + M} \quad \text{if } r_i > R, \end{aligned}\right\} \tag{11}$$

and finally,

$$f(r) = \sum_{r_i \geq r} \gamma_i = L(r) . \tag{12}$$

This estimate, for $r \geq o$, contains the unknown γ_o in the constant factor $1 - \gamma_o$, which does not affect the shape of the curve.

Reference: 1. C. F. Kent, Processing Detection Data Gathered from Targets of Opportunity, OEG IRM-7, 29 Dec 1961